THE 30th ANNUAL (2008) UNIVERSITY OF MARYLAND

HIGH SCHOOL MATHEMATICS COMPETITION

PART I MULTIPLE CHOICE

For each of the following 25 questions, carefully blacken the appropriate box on the answer sheet with a #2 pencil. Do not fold, bend or write stray marks on either side of the answer sheet. Each correct answer is worth 4 points. **Two points are deducted for each incorrect answer.** *Zero points are given if no box, or more than one box, is marked. Note that wild guessing is apt to lower your score. When the exam is over, give your answer sheet to the proctor. You may keep your copy of the questions.*

NO CALCULATORS

75 MINUTES

1. In 7 years, Mike will be 4 times as old as his dog Toby. Today, the sum of their ages is 71. How old is Mike?
   a. 29 b. 39 c. 56 d. 61 e. 63

2. This multiple-choice test has 25 questions. A student receives 4 points for a correct answer, loses 2 points for a wrong answer, and receives no points for an unanswered question. Randy randomly answers all the questions and gets 5 correct answers. Bianca answers 3 questions correctly, answers 3 questions incorrectly, and leaves the remaining 19 questions unanswered. Randy's score minus Bianca's score is
   a. 20 b. 12 c. 0 d. -12 e. -26

3. You are 5.5 feet tall. In the middle of the afternoon your shadow is 4 feet long. You pet giraffe's shadow is 11.2 feet long. How tall is your giraffe?
   a. 8.0 feet b. 8.96 feet c. 9.7 feet d. 14.2 feet e. 15.4 feet

4. For which real numbers a does the equation $ax^2 + 2x - 1 = 0$ have no real roots?
   a. $a < -1$ b. $a \leq -1$ c. $a > 0$ d. $a \geq 1$ e. $a \geq 2$

5. You have some coins in your pocket. You discover that you cannot choose a set of these coins to get a total of exactly \$1.00. What is the largest possible total value of the coins in your pocket? Coins can be worth \$0.01, \$0.05, \$0.10, \$0.25.
   a. \$0.99 b. \$1.09 c. \$1.19 d. \$1.25 e. \$1.31

6. Let $x = (2^{\log_2(3)})^{\log_3(4)}$. Then $x =$
   a. 0 b. 1 c. 2 d. 3 e. 4

7. In quadrilateral $ABCD$, diagonal $\overline{BD}$ is perpendicular to sides $\overline{AD}$ and $\overline{BC}$. The length of $\overline{BD}$ is 8. The length of side $\overline{AB}$ is 17 and the length of side $\overline{CD}$ is 10. What is the area of $ABCD$?
   a. 80 b. 84 c. 125 d. 144 e. 168

8. You have 200 coins. You give them to your friends in such a way that each friend gets at least one coin and no two friends get the same number of coins. What is the largest number of friends that you could have?
   a. 16 b. 17 c. 19 d. 20 e. 23

9. How many triples of real numbers (x, y, z) are solutions of the system of two equations

$$x + y = 2,\ xy - z^2 = 1\ ?$$

a. 0 b. 1 c. 2 d. 3 e. infinitely many

10. A man traveled from A to B at 40 miles an hour and then from B to A at 60 miles an hour. What was his average speed (in miles per hour) during the entire journey?
a. 46 b. 48 c. 50 d. 52 e. 54

11. What is the largest number that can be obtained as the product of two or more positive integers that add up to 20?
a. 625 b. 1024 c. 1458 d. 1998 e. 2008

12. How many numbers x with $10 \le x \le 99$ are 18 more than the sum of their digits?
a. 9 b. 10 c. 12 d. 18 e. 19

13. Stu lives on the top floor of an apartment building and Hal lives on a lower floor. Hal's window is at one fourth the height of Stu's window. Stu drops a stone out of his window. Three seconds later, Hal drops a stone out of his window. The two stones hit the ground at the same time. How high is Stu's window from the ground? The distance that an object falls in t seconds is $16t^2$ feet.
a. 144 feet b. 180 feet c. 520 feet d. 576 feet e. 720 feet

14. Let D be the number of digits (in base 10) of the number 2^{100}. Then
a. $D \le 16$ b. $17 \le D \le 20$ c. $21 \le D \le 29$ d. $30 \le D \le 35$ e. $36 \le D$

15. Alice and Bob live in an apartment building. Each floor of the building has ten apartments numbered by consecutive positive integers. Apartments 10 through 19 are on the first floor, apartments 20 through 29 are on the second floor, apartments 30 through 39 are on the third floor, and so on. It turns out that Bob lives on the n^{th} floor, where n is Alice's apartment number, and that the sum of Alice's and Bob's apartment numbers is 260. What is the difference of their apartment numbers (that is, Bob's number minus Alice's number)?
a. 0 b. 195 c. 214 d. 246 e. 310

16. Let $x = \sin 1$, $y = \sin 2$, $z = \sin 3$ (angles are in radians) Which of the following is true?
a. $x < z < y$ b. $x < y < z$ c. $y < x < z$ d. $y < z < x$ e. $z < x < y$

17. Program I takes a problem of size n and solves it in time $\log_2 \log_2 n$ steps ($\log_2$ means log base 2) *rounded up to the nearest integer.* Program II takes a problem of size n and solves it in time $\frac{1}{2}\log_2 n$ *rounded down to the nearest integer.* What is the largest integer n for which program II is faster than program I?
a. 63 b. 127 c. 128 d. 129 e. 255

18. A set X is *cool* if (1) $X \subseteq \{1, \ldots, 2008\}$, (2) X is nonempty, and (3) for all *distinct* $a, b \in X$, we have $a + b \in X$. How many cool sets are there?
a. 502 b. 1004 c. 2008 d. 3006 e. 4016

19. Let $P(X) = X^4 + aX^3 + bX^2 + cX + d$ be a polynomial, where a, b, c, d are integers. Suppose $P(0) = 2006$, $P(1) = 2007$, $P(2) = 2008$. What is the smallest possible positive value of $P(3)$?
a. 2 b. 5 c. 11 d. 2009 e. 2010

20. For each natural number n, let $S(n)$ denote the sum of the digits of n. Then the value of $S(1) + S(2) + \cdots + S(2008)$ is
a. 28036 b. 28054 c. 28072 d. 28081 e. 28090

21. Five probabilists at a meeting remove their name tags and put them in a bag. Then they each randomly choose one of the tags. What is the probability that exactly one of them gets the correct name tag?
a. 1/120 b. 1/5 c. 1/4 d. 3/8 e. 1/3

22. How many solutions in real numbers x does the equation

$$\sqrt{1 + \sqrt{x}} = x - 1$$

have?
a. 0 b. 1 c. 2 d. 3 e. 4

23. Cal and Tex use their calculators to calculate $\cos x$. Cal enters x as x degrees, while Tex enters x radians. Cal gets an answer y, while Tex gets the value $-y$. Let x the smallest positive value for which this happens. Then $x =$
a. 3 b. $\pi - 3$ c. $180 - \pi$ d. $\pi^2/180$ e. $180\pi/(180 + \pi)$

24. A bug lives on a corner of a cube and is allowed to travel only on the edges of the cube. In how many ways can the bug visit each of the other seven corners once and only once, returning to its home corner only at the end of the trip?
a. 6 b. 8 c. 12 d. 24 e. 27

25. How many real numbers x with $0 \leq x < 2\pi$ satisfy $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$? (angles are in radians)
a. 1 b. 3 c. 5 d. 7 e. 9