

THE 33rd ANNUAL (2011) UNIVERSITY OF MARYLAND
HIGH SCHOOL MATHEMATICS COMPETITION
PART I MULTIPLE CHOICE

For each of the following 25 questions, carefully blacken the appropriate box on the answer sheet with a #2 pencil. Do not fold, bend or write stray marks on either side of the answer sheet. Each correct answer is worth 4 points. **Two points are deducted for each incorrect answer.** Zero points are given if no box, or more than one box, is marked. Note that wild guessing is apt to lower your score. When the exam is over, give your answer sheet to the proctor. You may keep your copy of the questions.

NO CALCULATORS

75 MINUTES

1. The U.S. Treasury mints a new series of 8 and 13 cent coins. The cashier at a local supermarket gave us change for one dollar using only these new coins. How many coins did we receive?
a. 8 b. 9 c. 10 d. 11 e. 12
2. On the show “You can win \$1000,” you are given the lines $y = 13x - 1$ and $y = 10x + b$. If you choose b so that the intersection of the two lines has y -coordinate equal to 1000, then you win. What value of b should you choose?
a. 230 b. 576 c. 580 d. 1010 e. 2011
3. A total of 2000 students take a test, and they each answer both Question 1 and Question 2. Question 1 is answered correctly by 1500 students and Question 2 is answered correctly by 1200 students. Exactly 100 students get both Question 1 and Question 2 wrong. How many students answered both questions correctly?
a. 300 b. 500 c. 700 d. 800 e. 1000
4. Consider two positive numbers such that their difference is 2 and their product is 10. The larger number is equal to which of the following?
a. $\sqrt{7} + 2$ b. $\sqrt{7} - 2$ c. $2\sqrt{7}$ d. $\sqrt{11} + 1$ e. $\sqrt{11} - 1$
5. With one game remaining in the season, the baseball team had won $1/10$ of their games. They won their last game, and thereby ended up winning $1/9$ of their games for the season. How many games did they play in the season?
a. 10 b. 36 c. 54 d. 72 e. 81
6. Let AM be a median of the triangle ABC . If $AM = MB$ and $\angle ABC = 40^\circ$, what is the value of $\angle ACB$?
a. 20° b. 30° c. 40° d. 50° e. 60°
7. A function f has the property that $f(xy) = f(x) + f(y)$ for any two numbers x and y . If $f(10) = 6$ and $f(20) = 10$, compute the value of $f(25)$.
a. 4 b. 6 c. 8 d. 10 e. 12
8. Estimate, to four significant digits, the value of $\log(\sqrt[3]{10})$ (where the logarithm is in base 10).
a. .3010 b. .3333 c. .4771 d. .5000 e. 3.0000

9. Which of the following is largest (all angles are in degrees)?
a. $\sin(45^\circ) + \cos(45^\circ)$ b. $\sin(60^\circ) + \cos(60^\circ)$ c. $\sin(90^\circ) + \cos(90^\circ)$ d. $\sin(120^\circ) + \cos(120^\circ)$
e. $\sin(135^\circ) + \cos(135^\circ)$
10. A rectangle has area a and a diagonal of length d . Which expression below gives the length of its perimeter?
a. $2\sqrt{d^2 + a}$ b. $\sqrt{2}(d + \sqrt{2a})$ c. $2\sqrt{2d^2 + a}$ d. $2d\sqrt{2a}$ e. $2\sqrt{d^2 + 2a}$
11. Anna, Bob, and Cathy spent their afternoon picking apples and oranges. Anna had twice as many apples as Bob, while Bob had twice as many apples as Cathy. Bob had three times as many oranges as Cathy, who had three times as many oranges as Anna. At the end of the day they had fewer than 250 pieces of fruit in total, and an equal positive number of apples and oranges. How many pieces of fruit did they pick?
a. 180 b. 182 c. 184 d. 186 e. 188
12. A person rowing a boat against the river current got from point A to point B in 4 hours. It took him 2 hours to get from point B to point A rowing along the current. How long would it take him to row the same distance as between A and B in a lake without a current ?
a. 2 h 30 min b. 2 h 40 min c. 3 h d. 3 h 15 min e. 3 h 20 min
13. A kind farmer puts out a plate consisting of 2011 kernels of corn. On the first day, one bird flies in and eats one kernel. On the second day, two birds fly in and eat one kernel each. On the third day, three birds eat one kernel each, and so on. On some fateful day, there are not enough kernels left for each bird. On that day, how many kernels are on the plate?
a. 55 b. 56 c. 57 d. 58 e. 59
14. At a costume party involving members of Little Red Riding Hood's family and Big Bad Wolves, a group of three, call them A, B, and C, gets in a discussion. The Hoods always tell the truth and the Wolves always lie. First, A says, "C is a Wolf." Then B says "A and C are Hoods." Finally, C says, "B is a Hood." Which of the following is true?
a. A, B, C are Wolves b. A, B, C are Hoods c. A and C are Wolves and B is a Hood d. A and B are Wolves and C is a Hood e. A is a Hood, and B and C are Wolves
15. Find a real number r such that the equation $|x + 12| + |x - 5| = r$ has infinitely many solutions in x .
a. 5 b. 7 c. 12 d. 17 e. There is no such real number r
16. An equilateral triangle ABC has area 1. Erect an (exterior) square on each side of ABC . The six vertices of these three squares that do not coincide with a vertex of ABC form a convex hexagon. What is the area of this hexagon?
a. $6\sqrt{3}$ b. $4\sqrt{3}$ c. $(10\sqrt{3})/3$ d. $4 + 2\sqrt{3}$ e. $4(1 + \sqrt{3})$
17. A number machine takes an input x and gives $2x + 1$ as output. After you apply the machine 12 times, starting from the integer n , the output is 827,391. Compute n .
a. 198 b. 199 c. 201 d. 202 e. 204
18. In the standard version of tic-tac-toe, two players X and O alternately fill in a 3×3 board with their symbols, with X moving first. The game stops when one of the two players has marked three of his symbols in the same row, column, or diagonal. Of the 126 distinct ways of filling the board with five X's and four O's, how many can be the end result of a valid game of tic-tac-toe?
a. 68 b. 90 c. 108 d. 114 e. 126

19. The number 416183 is equal to the product pq of two prime numbers p and q whose sum satisfies $1292 \leq p + q \leq 1300$. Which of the following equals $p + q$?
- a. 1292 b. 1294 c. 1296 d. 1298 e. 1300

20. How many real numbers x are solutions of the equation $3^{2x} - 34 \cdot 15^{x-1} + 5^{2x} = 0$?
- a. 0 b. 1 c. 2 d. 3 e. 4

21. A, B, C, D, E, F , and G are distinct nonnegative integers that are less than 10. We have that

$$A - B = C \div D = E \times F = E + G = F.$$

Find the sum $A + B + C + D + E + F + G$.

- a. 31 b. 32 c. 33 d. 34 e. 35
22. How many triples (a, b, c) of positive integers satisfy both $a < b < c$ and $a + b + c = 1001$?
- a. 82750 b. 83000 c. 83250 d. 498000 e. 499500
23. Let x_1, \dots, x_n be all solutions of the equation

$$(\sin 3x)(\sin 3x - \cos x) = (\sin x)(\sin x - \cos 3x)$$

with $0 \leq x_i < 2\pi$ for each i . Which of the following gives the sum $x_1 + \dots + x_n$?

- a. 3π b. $7\pi/2$ c. $11\pi/2$ d. $13\pi/2$ e. 7π
24. The number '7abcdefg77' is a 10-digit number (written in base 10) that is equal to the cube of an integer. What is the digit g?
- a. 1 b. 2 c. 4 d. 7 e. 9
25. A teacher writes the numbers 1 and 2 on the blackboard. Each student, one at a time, goes to the board, chooses two numbers x and y that are written on the board (possibly $x = y$), computes $x + y + xy$, and writes the result on the board. When the students are finished, there are several numbers on the board, including 1727, 2303, 2467, 3455, and 7775. Unfortunately, one of these numbers was produced by a student using a calculator whose batteries were weak. Which of the five numbers *cannot* be produced by correct calculations by the students?
- a. 1727 b. 2303 c. 2467 d. 3455 e. 7775