THE 34$^{\text{th}}$ ANNUAL (2012) UNIVERSITY OF MARYLAND

HIGH SCHOOL MATHEMATICS COMPETITION

PART I MULTIPLE CHOICE

*For each of the following 25 questions, carefully blacken the appropriate box on the answer sheet with a #2 pencil. Do not fold, bend or write stray marks on either side of the answer sheet. Each correct answer is worth 4 points.* **Two points are deducted for each incorrect answer.** *Zero points are given if no box, or more than one box, is marked. Note that wild guessing is apt to lower your score. When the exam is over, give your answer sheet to the proctor. You may keep your copy of the questions.*

**NO CALCULATORS**

**75 MINUTES**

1. A department store is selling turkey costumes for \$200 each. After Halloween, the costumes go on sale at 10% off. After Thanksgiving, the sale is extended by a further 20% off the discounted price. What is the price of a turkey costume after the second reduction?
a. \$140  b. \$144  c. \$150  d. \$168  e. \$170

2. An integer $p$ is prime if $p > 1$ and the only positive divisors of $p$ are 1 and $p$. How many 2-digit prime numbers $p$ are there such that each digit of $p$ is also a prime?
a. 3  b. 4  c. 5  d. 6  e. 7

3. What is the least integer $n$ such that $|n - 2012| < 34$?
a. 1977  b. 1978  c. 1979  d. 1980  e. 2012

4. What is the value of $\dfrac{1}{1+\frac{1}{1+\frac{1}{1+1}}}$ ?
a. $\frac{4}{5}$  b. $\frac{3}{4}$  c. $\frac{2}{3}$  d. $\frac{3}{5}$  e. $\frac{\sqrt{5}-1}{2}$

5. Each license plate in the state of Ninea displays a 9-digit number which is a multiple of 9. After a bank robbery, the teller noticed that the license plate of the getaway car was $2012d2012$, where $d$ was an illegible digit. Determine $d$.
a. 1  b. 2  c. 5  d. 6  e. 8

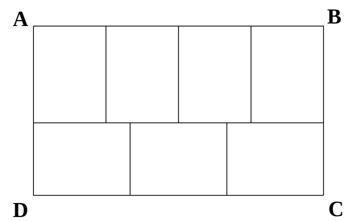
6. An isosceles right triangle has legs of length $4^{10}$. How long is its hypotenuse?
a. $4^{10.25}$  b. $4^{10.5}$  c. $4^{10.75}$  d. $4^{11}$  e. $4^{11.25}$

7. The parabola $y = ax^2 - 1$ intersects the coordinate $x$- and $y$-axes in three points which form the vertices of an equilateral triangle. Determine the value of $a$.
a. 1  b. $\sqrt{3}$  c. 2  d. $2\sqrt{3}$  e. 3

8. During a period of days, we observed that when it rained in the afternoon, it had been clear in the morning, and when it rained in the morning, it was clear in the afternoon. It rained on 9 days, and it was clear on 6 afternoons and 7 mornings. How long was this period?
a. 16 days  b. 14 days  c. 13 days  d. 12 days  e. 11 days

9. What is the last digit of $2012^{2013}$?
a. 2  b. 4  c. 6  d. 8  e. 0

10. The rectangle $ABCD$ is divided into seven smaller and equal rectangles, as in the figure. If the area of $ABCD$ is equal to 336, what is the perimeter of the rectangle $ABCD$?
a. 76 b. 86 c. 96 d. 106 e. 116




11. Compute $1000^{\log 2 + \log 3}$, where the logarithms are base 10.
a. 18 b. 125 c. 216 d. 5000 e. 6000

12. Three runners, A, B, and C, ran a 100 meter race. Each runner ran with a constant speed. When A finished, B was 10 meters behind him. When B finished, C was 10 meters behind him. What was the distance in meters between A and C when A crossed the 50 meter mark?
a. 9 b. 9.5 c. 10 d. 10.5 e. 11

13. Order $\tan 1$, $\tan 2$, and $\tan 3$ from smallest to largest (the angles are measured in radians).
a. $\tan 1 < \tan 2 < \tan 3$ b. $\tan 1 < \tan 3 < \tan 2$ c. $\tan 2 < \tan 3 < \tan 1$ d. $\tan 2 < \tan 1 < \tan 3$ e. $\tan 3 < \tan 2 < \tan 1$

14. Archimedes, Boole, Cauchy, Descartes, Euler, and Fermat agree to review a collection of math Ph.D. theses. They agree that everyone will read exactly 102 of them and that each thesis will be read by exactly 4 people. How many theses are there?
a. 153 b. 204 c. 306 d. 408 e. 612

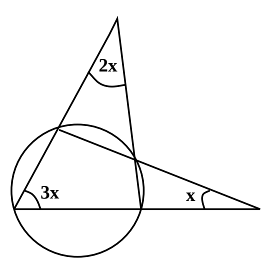
15. A *banana string* is a permutation of the six letters which appear in the word BANANA. For example, NNABAA and ANABAN are two banana strings. How many banana strings are there in total?
a. 30 b. 60 c. 80 d. 90 e. 120

16. Determine the length of the largest interval on which the function

$$f(x) = |x-1| + |x-2| + |x-3| + |x-4|$$

is constant. (A function $f$ is constant on an interval $[a, b]$ if $f(x) = f(y)$ for all $x, y \in [a, b]$).
a. 0 b. 1 c. 2 d. 3 e. 4

17. What is the angle $x$ in the picture below?
a. $10°$ b. $15°$ c. $18°$ d. $20°$ e. $25°$



18. Determine the least natural number which can be written both as a sum of 9 consecutive positive integers and as a sum of 10 consecutive positive integers.
a. 45 b. 55 c. 100 d. 135 e. 495

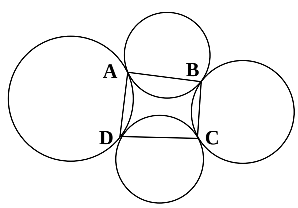
19. One of the five children baked a cake for their mother. Adam said: 'It was either Bob or Charlie'. Bob said: 'It was neither me nor Ethan'. Charlie said: 'You are both lying'. David said: 'No, one of them told the truth and the other one lied'. Ethan said: 'No, David, you are wrong'. The mother knows that three of her children always tell the truth. Who baked the cake?
a. Adam b. Bob c. Charlie d. David e. Ethan

20. If $r^2 - r - 10 = 0$ then which statement below is true about $(r+1)(r+2)(r-4)$ ?
a. It is an integer b. It is irrational and positive c. It is irrational and negative d. It is rational but not an integer e. It is not a real number

21. Let $f(x) = 1 - x + x^2 - \cdots - x^9 + x^{10}$. Substitute $x = y + 1$ in $f(x)$ to obtain a polynomial $g(y) = a_0 + a_1 y + \cdots + a_{10} y^{10}$ for some integer coefficients $a_i$. Compute $a_0 + a_1 + \cdots + a_{10}$.
a. 1 b. 255 c. 517 d. 683 e. 1025

22. What is the smallest integer $n$ such that $100!/(18)^n$ is not an integer?
a. 24 b. 25 c. 26 d. 27 e. 28

23. Four circles are tangent to each other at the points $A, B, C, D$, as shown in the figure. Which of the following statements is true about the quadrilateral $ABCD$?
a. There is a circle passing through all four vertices of $ABCD$ b. There is a circle tangent to all four sides of $ABCD$ c. The diagonals of $ABCD$ bisect each other d. The diagonals of $ABCD$ are perpendicular to each other e. The diagonals of $ABCD$ intersect at the center of mass of $ABCD$




24. Evaluate $\cos(20°) - \cos(40°) + \cos(60°) - \cos(80°)$.
a. $\sqrt{3}/2$ b. $\sqrt{3}/3$ c. $1/3$ d. $2/3$ e. $1/2$

25. A tournament took place between 5 chess players. Every player played exactly one game with each of the other 4 players. A player gets 1 point for a win and 1/2 a point for a draw. It is known that at the end of the event, no two players had the same number of points, the player in first place didn't have a single draw, the one in second place didn't lose a single game, and the one in fourth place didn't win a single game. How many games ended in a draw?
a. 2 b. 3 c. 4 d. 5 e. 6