30th ANNUAL UNIVERSITY OF MARYLAND
HIGH SCHOOL MATHEMATICS COMPETITION
PART II
December 3, 2008, 1:00–3:00

NO CALCULATORS
2 hours

1. Show that for every $n \geq 6$, a square in the plane may be divided into n smaller squares, not necessarily all of the same size.

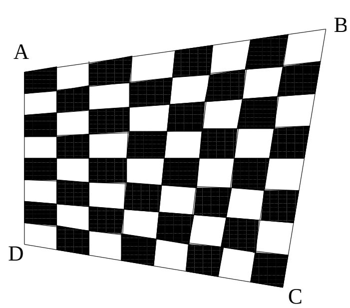
2. Let n be the 4018-digit number $111 \cdots 11222 \cdots 2225$, where there are 2008 ones and 2009 twos. Prove that n is a perfect square. (Giving the square root of n is not sufficient. You must also prove that its square is n.)

3. Let n be a positive integer. A game is played as follows. The game begins with n stones on the table. The two players, denoted Player I and Player II (Player I goes first), alternate in removing from the table a nonzero square number of stones. (For example, if $n = 26$ then in the first turn Player I can remove 1 or 4 or 9 or 16 or 25 stones.) The player who takes the last stone wins. Determine if the following sentence is TRUE or FALSE and prove your answer:

   > There are infinitely many starting values n such that Player II has a winning strategy.

   (Saying that Player II has a winning strategy means that no matter how Player I plays, Player II can respond with moves that lead to a win for Player II.)

4. Consider a convex quadrilateral $ABCD$. Divide side AB into 8 equal segments AP_1, P_1P_2, $\ldots$, P_7B. Divide side DC into 8 equal segments DQ_1, Q_1Q_2, $\ldots$, Q_7C. Similarly, divide each of sides AD and BC into 8 equal segments. Draw lines to form an 8×8 "checkerboard" as shown in the picture. Color the squares alternately black and white.
   (a) Show that each of the 7 interior lines P_iQ_i is divided into 8 equal segments.
   (b) Show that the total area of the black regions equals the total area of the white regions.



5. Prove that exactly one of the following two statements is true:
   A. There is a power of 10 that has exactly 2008 digits in base 2.
   B. There is a power of 10 that has exactly 2008 digits in base 5.