36$^{\text{th}}$ ANNUAL UNIVERSITY OF MARYLAND
HIGH SCHOOL MATHEMATICS COMPETITION
PART II
December 3, 2014, 1:00–3:00

NO CALCULATORS
2 hours

1. A *multimagic square* is a 3×3 array of distinct positive integers with the property that the product of the 3 numbers in each row, each column, and each of the two diagonals of the array is always the same.

   (a) Prove that the numbers 1, 2, 3, ..., 9 cannot be used to form a multimagic square.

   (b) Give an example of a multimagic square.

2. A sequence $a_1, a_2, a_3, \ldots, a_n$ of real numbers is called an *arithmetic progression* if

$$a_1 - a_2 = a_2 - a_3 = \cdots = a_{n-1} - a_n.$$

   Prove that there exist distinct positive integers $n_1, n_2, n_3, \ldots, n_{2014}$ such that

$$\frac{1}{n_1}, \frac{1}{n_2}, \ldots, \frac{1}{n_{2014}}$$

   is an arithmetic progression.

3. Let $\lfloor x \rfloor$ be the largest integer that is less than or equal to x. For example, $\lfloor 3.9 \rfloor = 3$ and $\lfloor 4 \rfloor = 4$. Determine (with proof) all real solutions of the equation

$$x^2 - 25\lfloor x \rfloor + 100 = 0.$$

4. An army has 10 cannons and 8 carts. Each cart can carry at most one cannon. It takes one day for a cart to cross the desert. What is the least number of days that it takes to get the cannons across the desert? (Cannons can be left part way and picked up later during the procedure.) Prove that the amount of time that your solution requires to move the cannons across the desert is the smallest possible.

5. Let C be a convex polygon with 4031 sides. Let p be the length of its perimeter and let d be the sum of the lengths of its diagonals. Show that

$$\frac{d}{p} > 2014.$$