38$^{\text{th}}$ ANNUAL UNIVERSITY OF MARYLAND
HIGH SCHOOL MATHEMATICS COMPETITION
PART II
November 30, 2016, 1:00–3:00

NO CALCULATORS
2 hours

1. Fill in each box with an integer from 1 to 9. Each number in the right column is the product of the numbers in its row, and each number in the bottom row is the product of the numbers in its column. Some numbers may be used more than once, and not every number from 1 to 9 is required to be used.

				70
				147
				512
				729
144	216	280	441	

2. A set X is called *prime-difference free* (henceforth pdf) if for all $x, y \in X$, $|x-y|$ is not prime. Find the number n such that the following both hold.
   - There is a pdf set of size n that is a subset of $\{1, \ldots, 2016\}$, and
   - There is no pdf set of size $n+1$ that is a subset of $\{1, \ldots, 2016\}$.

3. Let $X_1, \ldots, X_{15}$ be a sequence of points in the $xy-$plane such that $X_1 = (10, 0)$ and $X_{15} = (0, 10)$. Prove that for some $i \in \{1, 2, \ldots, 14\}$, the midpoint of $X_i X_{i+1}$ is of distance greater than $1/2$ from the origin.

4. Suppose that $s_1, s_2, \ldots, s_{84}$ is a sequence of letters from the set $\{A, B, C\}$ such that every four-letter sequence from $\{A, B, C\}$ occurs exactly once as a consecutive subsequence $s_k, s_{k+1}, s_{k+2}, s_{k+3}$. Suppose that $(s_1, s_2, s_3, s_4, s_5) = (A, B, B, C, A)$. What is s_{84}? Prove your answer.

5. Determine (with proof) whether or not there exists a sequence of positive real numbers $a_1, a_2, a_3, \ldots$ with both of the following properties:
   - $\sum\limits_{i=1}^{n} a_i \leq n^2$, for all $n \geq 1$, and
   - $\sum\limits_{i=1}^{n} \dfrac{1}{a_i} \leq 2016$, for all $n \geq 1$.