40th ANNUAL UNIVERSITY OF MARYLAND
HIGH SCHOOL MATHEMATICS COMPETITION
PART II
November 28, 2018, 1:00–3:00

NO CALCULATORS
2 hours

1. I have 6 envelopes full of money. The amounts (in dollars) in the 6 envelopes are six consecutive integers. I give you one of the envelopes. The total amount in the remaining 5 envelopes is $2018. How much money did I give you?

2. Two tangents AB and AC are drawn to a circle from an exterior point A. Let D and E be the midpoints of the line segments AB and AC. Prove that the line DE does not intersect the circle.

3. Let $n \geq 2$ be an integer. A subset S of $\{0, 1, \ldots, n-2\}$ is said to be *closed* whenever it satisfies all of the following properties:

   - $0 \in S$
   - If $x \in S$ then $n - 2 - x \in S$
   - If $x \in S, y \geq 0$, and $y + 1$ divides $x + 1$ then $y \in S$.

   Prove that $\{0, 1, \ldots, n-2\}$ is the only closed subset if and only if n is prime.

   (Note: "$\in$" means "belongs to".)

4. Consider the 3×3 grid shown below:

A	B	C
D	E	F
G	H	I

   A *knight move* is a pair of elements (s, t) from $\{A, B, C, D, E, F, G, H, I\}$ such that s can be reached from t by moving either two spaces horizontally and one space vertically, or by moving one space horizontally and two spaces vertically. (For example, (B, I) is a knight move, but (G, E) is not.) A *knight path of length n* is a sequence $s_0, s_1, s_2, \ldots, s_n$ drawn from the set $\{A, B, C, D, E, F, G, H, I\}$ (with repetitions allowed) such that each pair (s_i, s_{i+1}) is a knight move.

   Let N be the total number of knight paths of length 2018 that begin at A and end at A. Let M be the total number of knight paths of length 2018 that begin at A and end at I. Compute the value $(N - M)$, with proof. (Your answer must be in simplified form and may not involve any summations.)

5. A strip is defined to be the region of the plane lying on or between two parallel lines. The width of the strip is the distance between the two lines. Consider a finite number of strips whose widths sum to a number $d < 1$, and let D be a circular closed disk of diameter 1. Prove or disprove: no matter how the strips are placed in the plane, they cannot entirely cover the disk D.