THE 26th ANNUAL (2004) UNIVERSITY OF MARYLAND

HIGH SCHOOL MATHEMATICS COMPETITION

PART I MULTIPLE CHOICE

For each of the following 25 questions, carefully blacken the appropriate box on the answer sheet with a #2 pencil. Do not fold, bend or write stray marks on either side of the answer sheet. Each correct answer is worth 4 points. **Two points are deducted for each incorrect answer.** *Zero points are given if no box, or more than one box, is marked. Note that wild guessing is apt to lower your score. When the exam is over, give your answer sheet to the proctor. You may keep your copy of the questions.*

NO CALCULATORS

75 MINUTES

1. Which number is the largest? (*Note:* a^{b^c} means $a^{(b^c)}$)
   a. 3^{2^1} b. 2^{1^3} c. 3^{1^2} d. 1^{2^3} e. 2^{2^2}

2. If $6x + 7y = 2004$ and $7x + 6y = 4002$, then $x + y =$
   a. 221 b. 400 c. 462 d. 487 e. 770

3. Let S be a square pizza of side s. Let C be a circular pizza of radius r. The pizzas S and C have the same perimeter. Let A_S be the area of S, and let A_C be the area of C. What is A_S/A_C?
   a. $\pi^2/4$ b. $4/\pi^2$ c. 1 d. $4/\pi$ e. $\pi/4$

4. SpongeBob SquareDigits likes 2-digit numbers $n \geq 10$ such that both digits are squares (for example, 10 and 41 are two such numbers). How many of these numbers are there?
   a. 3 b. 6 c. 8 d. 12 e. 15

5. There are 5 geese in a gaggle. If, working together, the gaggle produces 55 eggs in 5555 days, what is the average number of days it takes a single goose to lay an egg?
   a. 11 b. 101 c. 505 d. 555 e. 1111

6. A chain costs \$1.00 per foot. A customer asks the merchant for 4 feet of chain. The merchant takes a 10-foot length of chain and cuts off a piece that he claims is 4 feet long. However, the customer notices that the piece of chain is only 44 inches long. Therefore the customer quickly announces that he instead wants 6 feet of chain and will take the other piece. He then pays \$6.00 to the merchant, who unhappily accepts the payment. How much (to the closest cent) does the customer pay per foot for the chain?
   a. \$0.76 b. \$0.83 c. \$0.93 d. \$0.95 e. \$0.97

7. Let x and y be real numbers. The smallest possible value of $|2 - x| + |x - y| + |y - 2004|$ is
   a. 2 b. 1003 c. 2002 d. 2004 e. 2006

8. If $0 < x < 90^\circ$ and $\sin x = 3/5$, then $\tan x =$
   a. 0.25 b. 0.45 c. 0.75 d. 0.8 e. 1.2

9. Fresh cherries contain 99% water. You start with 100 pounds of fresh cherries. After a few hours in the sun, some of the water evaporates and the percentage of water in the cherries becomes 98%. What is the new weight of the cherries?
   a. 90 pounds b. 50 pounds c. 80 pounds d. 95 pounds e. 98 pounds

10. An ancient manuscript had the statement that $X^2+bX+30$ has two integer roots. Unfortunately, it was impossible to read the positive integer b. How many possibilities are there for b?
    a. none b. 2 c. 3 d. 4 e. 6

11. Which of the following is the largest?
    a. $2^{\log_5 6}$ b. $2^{\log_6 5}$ c. $3^{\log_6 5}$ d. $3^{\log_5 6}$ e. 3

12. Let $a = \sqrt{\sqrt{26} + \sqrt[3]{65}}$, $b = \sqrt{\sqrt[3]{26} + \sqrt{65}}$, and $c = \sqrt[3]{\sqrt{26} + \sqrt{65}}$. Which of the following is true?
    a. $a < b < c$ b. $b < a < c$ c. $c < b < a$ d. $a < c < b$ e. $c < a < b$

13. Exactly one of the three statements A, B, C is true and the other two are false. Suppose exactly one of the following statements a, b, c, d, e below is true. Which is it?
    a. B is true b. A is false and B is true c. C is false and A is true d. C is true e. B is false and A is true

14. The admission fee to a mathematics exhibition was \$15. When the fee was reduced, the (nonzero) number of customers per day went up by 50%, and the amount of money collected per day went up by 25%. What was the reduced fee?
    a. \$10.98 b. \$12.50 c. \$13.57 d. \$14.16 e. \$14.25

15. A stock broker trades stocks during 100 days. On day 1 he loses \$100. Every day after that he loses \$100 more than on the previous day. The average daily loss during the 100 day period is
    a. 5050 b. 10000 c. 100 d. 252525 e. 50000

16. The polynomial $X^2 - 9X + 3$ has roots r and s. If $X^2 + bX + c$ has roots r^2 and s^2, then (b, c) is
    a. $(-82, 9)$ b. (200, 4) c. (100,9) d. (12, 15) e. $(-75, 9)$

17. What is the last digit in the decimal expansion of $7^{(7^7)}$?
    a. 1 b. 3 c. 5 d. 7 e. 9

18. Find the area of a regular octagon inscribed into a circle of radius 1.
    a. $\pi/2$ b. $\sqrt{2}$ c. $2\sqrt{3}$ d. $5/2$ e. $2\sqrt{2}$

19. Two hikers started walking at noon, the 1st from A to B, the 2nd from B to A along the same path. They each walked at a constant speed. They met at 3 pm. The first came to B 2.5 hours before the 2nd got to A. When did the second hiker get to A?
    a. 6:00 b. 6:30 c. 7:00 d. 7:30 e. 8:00

20. Peter Rabbit starts hopping at the point (0,0) and finishes at (6,0). Each hop is from a point of the form (x, y) to one of the points $(x + 1, y \pm 1)$. However, he is not allowed to jump on a point (x, y) with $y < 0$ (because there is poison ivy in the fourth quadrant). How many paths are there from (0,0) to (6,0)?
a. 5 b. 6 c. 7 d. 8 e. 9

21. Let k be the largest positive integer, not a multiple of 10, such that removing the last 2 digits of k^2 leaves a perfect square. Then
a. $0 < k < 20$ b. $20 < k < 50$ c. $50 < k < 100$ d. $100 < k < 200$ e. $200 < k$

22. How many values of A are there such that the system of equations $x^2 = y^2$, $(x - A)^2 + y^2 = 1$ has exactly 3 solutions (x, y)?
a. 0 b. 1 c. 2 d. 3 e. 4

23. The Seven Dwarfs write down all 7-digit numbers that can be formed using each of the digits 1, 2, 3, 4, 5, 6, 7 exactly once (for example, 3175426 is one of their numbers). The Wicked Witch, using the phrase "divide and conquer" literally, looks for pairs (a, b) of such numbers so that b is a multiple of a but $a \neq b$. How many pairs are there?
a. 0 b. 1 c. 2 d. 3 e. 4

24. Which of the three statements are true? (i) There exist two triangles such that the sides of one triangle are all less than 1 inch while the sides of the other triangle are all bigger than 100 feet but the area of the first triangle is bigger than the area of the second. (ii) There exists a triangle whose altitudes are all shorter than 1 inch but whose area is bigger than 100 square inches.
(iii) There exists a triangle whose altitudes are all longer than 2 inches but whose area is less than 1 square inch.
a. (i) b. none c. (i) and (ii) d. (i) and (iii) e. (ii) and (iii)

25. Let $m = 1111 \cdots 111$ (2004 ones) and $n = 1111 \cdots 111$ (666 ones). The greatest common divisor of m and n is
a. 111 b. 333 c. 111111 (6 ones) d. 333333 e. 111111111111 (12 ones)