

THE 27th ANNUAL (2005) UNIVERSITY OF MARYLAND
HIGH SCHOOL MATHEMATICS COMPETITION
PART I MULTIPLE CHOICE

For each of the following 25 questions, carefully blacken the appropriate box on the answer sheet with a #2 pencil. Do not fold, bend or write stray marks on either side of the answer sheet. Each correct answer is worth 4 points. **Two points are deducted for each incorrect answer.** Zero points are given if no box, or more than one box, is marked. Note that wild guessing is apt to lower your score. When the exam is over, give your answer sheet to the proctor. You may keep your copy of the questions.

NO CALCULATORS

75 MINUTES

1. There are several dogs and people in a dog park. A cat goes by, is barked at by the dogs, and notes that there are 82 feet and 26 heads (the cat is not included). How many dogs are there?
a. 8 b. 10 c. 12 d. 15 e. 18
2. Let T be the set of points in the xy -plane satisfying $|x| + |y| \leq 4$. The area of T is
a. 4 b. 8 c. 12 d. 16 e. 32
3. Which of the following numbers is the largest?
a. 1^1 b. $2^{1/2}$ c. $3^{1/3}$ d. $4^{1/4}$ e. $5^{1/5}$
4. If 100 wolves can devour 50 sheep in 2 weeks, how many sheep can 50 wolves eat in 50 weeks?
a. 50 b. 100 c. 500 d. 625 e. 2500
5. A dinosaur weighs 65536 pounds at the beginning of the year. During January, its weight increases by 50%. Because dust from a meteorite impact blocks the sunlight during February, its weight decreases by 50% during that month. But the sunlight is back and the dinosaur is hungry during March, so its weight increases by 50% during that month. During April it has a stomach ache from eating too much, so its weight decreases by 50%. It continues alternately gaining 50% and losing 50% of its weight for the remaining eight months of the year. How many pounds does the dinosaur weigh at the end of the year?
a. 11664 b. 23328 c. 32768 d. 65536 e. 131072
6. A parent has a model car that is identical in proportions to the family car, and its length is $1/20$ the length of the family car. The parent carefully paints the model car, using 1 ounce of paint. The child, inspired by the parent, goes outside and paints the family car. How many ounces of paint does the child use? (Assume that the thickness of the paint is the same and that the child does not spill any cans of paint.)
a. 20 b. 40 c. 200 d. 400 e. 8000
7. The top of a rectangular box has area 120 square inches, the front has area 96 square inches, and the side has area 80 square inches. How high is the box?
a. 8 b. 10 c. 12 d. 15 e. 24
8. Which of the following numbers is the largest?
a. 10^{100} b. 20^{80} c. 30^{60} d. 40^{40} e. 50^{20}

9. If $\sin \theta = 3/7$, then

$$\sin^2 \theta + \frac{1}{1 + \tan^2 \theta}$$

is which of the following?

- a. $4/7$ b. $16/49$ c. $3/10$ d. $3/2$ e. 1

10. What is the value of the following (logarithms are to base 10)?

$$\log(2/1) + \log(3/2) + \log(4/3) + \cdots + \log(99/98) + \log(100/99)$$

- a. 1 b. 2 c. 10 d. 20 e. 100

11. A math class has between 15 and 40 students. Exactly 25% of the class knows how to play poker. On a certain Wednesday, 7 students were absent (because they were participating in a math contest). On that day, exactly 20% of the students attending the class knew how to play poker. How many students attending the class on that day knew how to play poker?

- a. 3 b. 4 c. 5 d. 8 e. 10

12. Some pigs have 2-looped tails and other pigs have 3-looped tails. While walking through a pig sty one day, you count exactly 100 loops. Based on this information, which of the following conclusions can you draw?

- a. The total number of pigs with 3-looped tails is odd. b. The total number of pigs is even.
c. The total number of pigs with 2-looped tails cannot be prime. d. The total number of pigs is odd. e. The total number of pigs with 3-looped tails is even.

13. What is the smallest altitude in the triangle with sides 3, 4 and 5?

- a. $5/7$ b. 1 c. 2 d. $12/5$ e. 3

14. The line $y = bx - b$ intersects the parabola $y = x^2$ in exactly one point (x, y) . If $b \neq 0$, what is the value of b ?

- a. 1 b. 2 c. 3 d. 4 e. 5

15. Equilateral triangle ABC has area 1. Extend side AB to a point X so that B is the midpoint of AX . Similarly, extend BC so that C is the midpoint of BY , and extend CA so that A is the midpoint of CZ . The area of triangle XYZ is

- a. 4 b. $4\sqrt{2}$ c. 7 d. $3\sqrt{3}$ e. 9

16. Space aliens decide to build another ring around Saturn. They draw a circle and place two rocks, each with mass 1 gram, that cut the circle into two half circles. Then the first alien looks at each half circle and puts a rock with mass 2 grams in the middle of each of the two arcs. The circle is thus divided into 4 quarter circles. For each of the four resulting arcs, the second alien adds up the masses of the rocks at the two ends of the arc and places a rock with that mass in the middle of the arc. The third alien then looks at each of the resulting 8 arcs, adds up the masses of the rocks that are at the two ends of the arc, and places a rock with that mass in the middle of the arc. They continue in this fashion, with each successive alien adding up the masses at the two ends of each arc and dividing the arc in two by placing a rock, with mass equal to this sum, in the middle of the arc. They stop as soon as the total mass of the rocks they have placed is greater than the mass of Saturn, namely 6×10^{29} grams. Let n be the number of aliens who placed rocks on the circle. Which of the following is true?

- a. $1 < n < 20$ b. $20 < n < 100$ c. $100 < n < 500$ d. $500 < n < 2005$ e. $2005 < n$

17. Suppose A, B, C, D are positive integers such that exactly one of the following inequalities is false. Which inequality is false?
 a. $A < B$ b. $C < D$ c. $A + C < B + C$ d. $A + C < B + D$ e. $A < B + C + D$

18. Among all rectangles of area 3, what is the smallest possible value of the sum of the lengths of its diagonals?
 a. $\sqrt{8}$ b. $2\sqrt{6}$ c. $2\sqrt{10}$ d. 8 e. 10

19. There are two positive integers $n \geq 3$ for which the expression

$$\frac{\log 2 \cdot \log 3 \cdot \log 4 \cdots \log n}{10^n}$$

takes on its smallest value. (Here \log stands for the logarithm to the base 10.) What is the larger of these two integers?

- a. $n = 4$ b. $n = 10^{10}$ c. $n = 10^{100}$ d. $n = 100^{10}$ e. $n = 2 \times 10^{100}$
20. Two handymen, Peter and Paul, agree to paint the interior of a house and lay hardwood floor. The total area of the walls to be painted is 4000 square feet and the total floor area is 1500 square feet. In an hour of hard work, Peter can either paint 40 square feet or lay 15 square feet of hardwood while Paul, a better hardwood layer, can either paint 20 square feet in an hour or lay 30 square feet of hardwood. What is the minimum time (in hours) in which Peter and Paul, working together, can finish the job?
 a. 60 b. 75 c. $83\frac{1}{3}$ d. $85\frac{2}{3}$ e. 100
21. The City Council has nine members, who serve on different committees. Each committee has three members and no two members serve together on more than one committee. Determine the largest possible number of committees.
 a. 9 b. 10 c. 12 d. 13 e. 24
22. There are 2005 people at a party. By the end of the party, in every group of four people there is at least one person who has shaken hands with the other three members of the group. What is the smallest possible number of people who have shaken hands with everyone at the party?
 a. 1002 b. 1003 c. 2001 d. 2002 e. 2003
23. A group of N islands are connected by bridges. Each island has bridges to at most 3 other islands. One can travel between any 2 islands by crossing at most two bridges. What is the largest possible value of N ? (Bridges are allowed to go over or under other bridges.)
 a. 7 b. 8 c. 9 d. 10 e. 11
24. Whenever trick-or-treaters come to her door on Halloween, a mathematician makes them choose two positive real numbers x and y . She lets s be the smallest of x , $y + 1/x$, and $1/y$ and gives them s pounds of candy. What is the largest possible value of s ?
 a. $\sqrt{2}$ b. 1 c. $3/2$ d. 2 e. none of the previous
25. After all twenty participants in a figure skating tournament skated, each of the 9 judges ordered the participants from place 1 (the best) to place 20 (the worst). It turned out that for each participant, the places assigned by different judges were not more than 3 apart. The sum of the places for each participant was calculated and the sums were ordered: $c_1 \leq c_2 \leq \dots \leq c_{20}$. What is the largest possible value of c_1 ?
 a. 18 b. 19 c. 21 d. 22 e. 24