

THE 25th ANNUAL (2003) UNIVERSITY OF MARYLAND
HIGH SCHOOL MATHEMATICS COMPETITION
PART I MULTIPLE CHOICE

For each of the following questions, carefully blacken the appropriate box on the answer sheet with a #2 pencil. Do not fold, bend or write stray marks on either side of the answer sheet. Each correct answer is worth 4 points. **Two points are deducted for each incorrect answer.** Zero points are given if no box, or more than one box, is marked. Note that wild guessing is apt to lower your score. When the exam is over, give your answer sheet to the proctor. You may keep your copy of the questions.

NO CALCULATORS

75 MINUTES

- Which of the following numbers is the largest?
a. $2^{(3^5)}$ b. $2^{(5^3)}$ c. $3^{(2^5)}$ d. $3^{(5^2)}$ e. $5^{(3^2)}$
- Batman and Robin each order a pizza. The circumference of Batman's pizza is 20% greater than the circumference of Robin's pizza. The area of Batman's pizza is what percentage greater than the area of Robin's?
a. 4 b. 10 c. 20 d. 40 e. 44
- The inaugural meeting of the local canine club was attended by 12 large, 5 medium and 8 small dogs. Of the 25 dogs, 15 were mixed breed ('mutts'), of which 5 were large, 5 were medium and 5 were small, while the other 10 dogs were pure breeds. How many large, pure breed dogs attended the meeting?
a. 5 b. 7 c. 9 d. 10 e. 12
- Which answer describes the set of all x such that $|3x + 4| > 13$?
a. $x > 3$ b. $x < -17/3$ or $x > 3$ c. $-3 < x < 3$ d. $x < -3$ or $x > 3$ e. $-3 \leq x \leq 3$
- Let $a \leq b \leq c$ be the lengths of the sides of a triangle T . If $a^2 + b^2 < c^2$ then which of the following must be true?
a. All 3 angles of T are acute. b. Some angle of T is obtuse. c. One angle of T is a right angle. d. T is equilateral. e. No such triangle can exist.
- For which x does $\log_2(8) - \log_3(9) = \log_5(x)$?
a. $1/5$ b. $4/5$ c. 1 d. 5 e. 125
- More than 93% of the students in a math class are girls, but there is at least one boy in the class. What is the smallest possible size of the class?
a. 13 b. 14 c. 15 d. 20 e. 21

8. The Beatles took a math exam. Paul got correct half of the questions plus 7 questions, John got correct one third of the questions plus 17 questions, George got correct one fourth of the questions plus 22 questions, and Ringo got correct one fifth of the questions plus 25 questions. There were between one and 100 questions on the exam and each Beatle got an integer number of questions correct. Which Beatle got the most questions correct?
- a. only Paul b. only John c. only George d. only Ringo e. at least two Beatles got the most questions correct.
9. Let C be a cube where the length (in inches) of its long diagonal is the same as its volume (in cubic inches). What is the length (in inches) of each side?
- a. 1 b. $2^{1/4}$ c. $3^{1/2}$ d. $2^{1/3}$ e. $3^{1/4}$
10. If θ is an acute angle with $\tan \theta = 1/3$, then $\sin \theta$ equals
- a. $1/\sqrt{3}$ b. $1/\sqrt{2}$ c. $1/2$ d. $1/\sqrt{10}$ e. $1/\pi$
11. Jack is older than Jill, but is less than twice as old. If r denotes the ratio of Jack's age to Jill's age today, then which formula represents the ratio of Jack's age to Jill's age when Jack's age was equal to Jill's age now?
- a. $1/r$ b. $\frac{r}{4-r^2}$ c. $\log_2(r)$ d. $\frac{1}{2-r}$ e. $\frac{r+1}{2-r}$
12. The value of $25^{1/\log 25}$ is which of the following? (\log is logarithm to the base 10)
- a. 1 b. 5 c. 10 d. 25 e. 125
13. Let S be a square of side s . Let C be a circle of radius r . Jim tells us that S and C have the same area and that the perimeter of S is the same as the circumference of C . What can we conclude?
- a. $s/r = 1$ b. $s/r = \sqrt{\pi}$ c. $s/r = \pi$ d. $s/r = \pi^2$ e. Jim is wrong
14. The numbers b, c, d are all integers. The parabola $y = x^2 + bx + c$ and the line $y = dx$ have exactly one point in common. Given these assumptions, which of the following statements is necessarily true?
- a. $b = 0$ b. $d - b$ is even c. $c = 0$ d. $|d| \geq |a|^2 + |b|^2$ e. $d > 1$
15. Suppose you are given the following three statements: 1. No kitten that loves fish has green eyes. 2. All kittens with whiskers love fish. 3. No kitten has a tail unless it has whiskers. Which of the following statements is a valid conclusion?
- a. No kitten with green eyes has a tail. b. All kittens with tails have green eyes.
c. No kitten that loves fish has a tail. d. All kittens that have tails have no whiskers.
e. All kittens with green eyes have tails.
16. Suppose x is a real number with $x > 1$. If $x^x = y$ and $y^y = 10^{2003}$, then
- a. $2 < x < 3$ b. $3 < x < 4$ c. $4 < x < 5$ d. $5 < x < 2000$ e. $2000 < x < 2003$

17. A student participated in a competition in which 20 problems were given. For each problem answered correctly the student received 8 points, but 5 points were deducted for each incorrectly answered problem. For a problem that he did not answer, 0 points were given. Given that the student's total score was 13 points, how many problems did the student submit answers for?
 a. 5 b. 7 c. 13 d. 15 e. 20
18. Given that $a + \frac{1}{a} = 3$, what is the value of $\left|a - \frac{1}{a}\right|$?
 a. $\sqrt{5}$ b. $\sqrt{2}$ c. 1.5 d. 2 e. $\sqrt[3]{3}$
19. The altitude to the hypotenuse of a right triangle divides the hypotenuse into two line segments, one of length 2003 and the other of length 25. What is the area of the triangle?
 a. $5070\sqrt{2003}$ b. $\frac{5\sqrt{2003}}{2}$ c. $\sqrt{2028}$ d. 50075 e. $5\sqrt{2003}$
20. The three little pigs are digging a moat to keep nasty wolves away. The first two pigs, working together, could dig the moat in two hours. The first and third pigs, working together, could dig the moat in one hour and twelve minutes. The second and third pigs, working together, could complete the job in an hour and a half. How long will it take all three pigs working together to dig the moat?
 a. 36 min b. 45 min c. 50 min d. 54 min e. one hour
21. It takes five hours for a steamship to travel downstream on a river from port A to port B and seven hours to make the same trip upstream from B to A. How long would it take for a raft, which is propelled only by the current of the river, to go from A to B?
 a. 63 hours b. 35 hours c. 12 hours d. 8 hours e. 7.2 hours
22. The stock of a small company is distributed among 2003 shareholders. It is known that any 1100 shareholders in the group possess at least half of the total number of shares of stock. What is the largest fraction of the total number of shares that can be held by any one stockholder?
 a. $9/100$ b. $1100/2003$ c. $903/2003$ d. $904/2003$ e. $25/3303$
23. How many four digit numbers $abcd$ (in base 10) are there with $a > b > c > d \geq 0$?
 a. 120 b. 144 c. 166 d. 210 e. 1066
24. How many 7-digit sequences of 0's and 1's are there that have a block of 3 successive 0's but do not have a block of 4 or more successive 0's?
 a. 8 b. 27 c. 28 d. 32 e. 50
25. A playing field is filled with 2001 Trolls, 2002 Griffins, and 2003 Dragons. Whenever two animals of different species shake hands, they both instantly disappear and are replaced by an animal of the third species. This game continues for some time, until there is only one species of animal left in the field. Which of the following is a possible end position of such a game?
 a. Exactly 5 Trolls remain. b. Exactly 25 Griffins remain. c. Exactly 9 Trolls remain.
 d. Exactly 4 Dragons remain. e. Exactly 6 Dragons remain.