

MATHEMATICS COMPETITION
UNIVERSITY OF MARYLAND
PART I MULTIPLE CHOICE
Fall 1997

For each of the following questions, carefully blacken the appropriate box on the answer sheet with a #2 pencil. Do not fold, bend or write stray marks on either side of the answer sheet. Each correct answer is worth 4 points. Two points are deducted for each incorrect answer. Zero points are given if no box, or more than one box, is marked. Note that wild guessing is apt to lower your score.

When the exam is over, give your answer sheet to the proctor. You may keep your copy of the questions.

**NO CALCULATORS
75 MINUTES**

1. Which number is the largest? a. 1^{44} b. 2^{33} c. 3^{22} d. 4^{11} e. 1,000,000,000
2. Pizzas are to be ordered for a party. The restaurant offers three specials.
Special A: a small, together with a medium pizza, for 13 dollars.
Special B: 2 medium pizzas for 14 dollars.
Special C: a large pizza for 15 dollars.
A small pizza has diameter 12", a medium pizza 16" and a large pizza 20".
Rank A, B and C from the greatest to the least square inches per dollar.
a. CBA b. BCA c. ABC d. ACB e. BAC
3. Many rectangular placemats have the following property: If folded in half along the shorter midline, the two sides of the resulting rectangle have the same ratio as in the original rectangle. Assuming that a placemat has this property, what is the ratio of its length to its width?
a. $\sqrt{2}$ b. 2 c. $\sqrt{3}$ d. 3 e. 4
4. Let \log denote logarithm to the base 10. The expression $5^{\log 2} + 2^{\log 5} - 50^{\log 2}$ equals
a. $5^{\log 2}$ b. 1 c. 25 d. $5^{\log 2 + \log 5}$ e. 0
5. What is the area of the region in the xy -plane defined by the inequalities $x \geq 0$, $y \geq x + 1$, and $2x + y \leq 10$?
a. 10 b. 12 c. $\frac{27}{2}$ d. $\frac{35}{2}$ e. 30

6. Several roosters want to buy an alarm clock. If each contributes \$0.35, they lack \$4.40. If each contributes \$0.40, they have \$4.40 extra. The number of roosters is in the range of
a. less than 50 b. 50 to 100 c. 100 to 150 d. 150 to 200 e. more than 200
7. How many numbers b are there among $\{1, 2, \dots, 100\}$ for which there is a positive integer a with $a^3 = b^2$? a. 1 b. 2 c. 3 d. 4 e. more than 4
8. Suppose that a flight from Washington to San Francisco takes 7 hours, while the flight from San Francisco to Washington takes 5 hours. The difference in time is due to a wind blowing from west to east. How long, in minutes, would it take to fly between the two cities if there were no wind? a. 330 b. 340 c. 350 d. 360 e. 375
9. Seven black unit squares and 2 red ones on a table are to be assembled into a 3×3 square, by matching sides. How many different designs can be made? Two designs are different when they look different no matter how you rotate them on the table. Flipping is forbidden. a. 7 b. 10 c. 12 d. 14 e. 17
10. Bubble gum sticks to a bike wheel of diameter 1 meter. Tom rode the bike for 1 kilometer without skidding. The number of times the bubble gum hit the ground is closest to which of the following numbers?
a. 290 b. 320 c. 350 d. 380 e. 410
11. The three vertices of a triangle are at $(0, 0)$, $(545, 0)$ and $(751, 915)$. The medians intersect at the point a. $(434, 304)$ b. $(433, 304)$ c. $(433, 305)$ d. $(432, 305)$ e. None of the preceding.
12. Street lamps come in three different colors. In how many ways can seven lamps be put in a row so that no neighboring lamps have same color?
a. 128 b. 192 c. 307 d. 343 e. 2184
13. Huey can weed Donald's garden in 20 minutes, Dewey can weed the garden in 24 minutes, and Louie can weed the garden in 30 minutes. In a spirit of cooperation they all decide to work together to weed the garden. How many minutes will it take them? a. 7 b. 8
c. 9 d. 10 e. 11

14. Two of six persons stole apples. But who? A said "B and C." D said "E and F." E said "A and B." B said "F and D." C said "B and E." F cannot be found. Four of the five persons interrogated had named one apple thief correctly and one incorrectly. The fifth had named both incorrectly. Who stole the apples? a. A and F b. C and E c. D and E d. B and D e. not enough information
15. Cut a round pizza by four straightline cuts. Moving pieces is not allowed between cuts. What is the largest possible number of pieces? a. 10 b. 11 c. 13 d. 16 e. 18
16. For how many different values of $k = 1, 2, 3, \dots$ does the k -th day of September fall on the same day of the week as the $2k$ -th day of October?
a. 0 b. 1 c. 2 d. 3 e. 4
17. It took 2322 digits to number the pages of a dictionary (all pages are numbered). How many pages are there in the dictionary?
a. 805 b. 810 c. 818 d. 823 e. None of the preceding.
18. How many values of x are there with $0 \leq x \leq \pi$ such that $2 \cos^2(x) = 1 - \cos(3x)$? (x is in radians.)
a. 1 b. 2 c. 3 d. 4 e. 5
19. The angle AYZ is 60° . Project A to line YZ perpendicularly to get point B, then project B to line YA perpendicularly to get point C. This procedure for point A is repeated for C to get D, E and so on. Denote T_1, T_2, T_3, \dots the triangles $ABY, BCY, CDY \dots$. Which is the first triangle with area smaller than one hundredth of T_1 ? a. T_5 b. T_{26} c. T_{27}
d. T_{50} e. T_{51}
20. Suppose a, b, c are three nonzero numbers and the polynomial $p(X) = X^3 - aX^2 + bX - c$ factors as $(X - a)(X - b)(X - c)$. What is the value $P(2)$?
a. -3 b. 0 c. 4 d. 7 e. 9
21. You are on a train moving at 80 km/h. An oncoming train of length 150 m passes by the window in 3 sec. How fast, in km/h, is the other train going?
a. 100 b. 93 c. 86 d. 79 e. None of the preceding.

22. Donald Duck read a book in 3 days. During the first day he read $1/5$ of the book, plus 16 pages. During the second day he read $3/10$ of what remained, plus 20 pages. During the third day he read $3/4$ of what remained, plus 30 pages. How many pages were there in the book?
a. 225 b. 240 c. 265 d. 270 e. None of the preceding.
23. Let $s(n)$ be the number of perfect squares with exactly n digits. Let $r = s(101)/s(100)$. Which of the following is correct?
a. $1.01 \leq r \leq 2.02$ b. $2.02 < r < 3$ c. $3 \leq r < 4$ d. $4 \leq r \leq 101$ e. $101 < r$
24. The largest real root of $X^4 - 4X^3 + 5X^2 - 4X + 1$ is
a. $2 + \sqrt{11} - \sqrt{2}$ b. $\sqrt{7}$ c. $\frac{5}{2}$ d. $\sqrt{2 + \sqrt{7}}$ e. $\frac{3 + \sqrt{5}}{2}$
25. An elephant is big and heavy. It has 4 legs and a trunk, 5 extremities altogether. A hippopotamus is also big and heavy. It has only 4 extremities, the legs. After he was given only the total number E of extremities in a herd of elephants and hippopotamuses, Tarzan managed to determine the number of elephants and the number of hippopotamuses in the herd and claimed that E was the largest possible number for which he could do it. How many elephants were in the herd?
a. 1 b. 2 c. 3 d. 4 e. 5

END OF THE EXAM