

MATHEMATICS COMPETITION  
UNIVERSITY OF MARYLAND  
PART I MULTIPLE CHOICE  
October 21, 1998

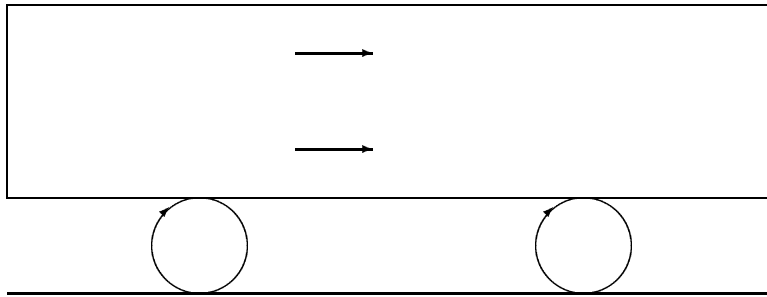
*For each of the following questions, carefully blacken the appropriate box on the answer sheet with a #2 pencil. Do not fold, bend or write stray marks on either side of the answer sheet. Each correct answer is worth 4 points. Two points are deducted for each incorrect answer. Zero points are given if no box, or more than one box, is marked. Note that wild guessing is apt to lower your score.*

*When the exam is over, give your answer sheet to the proctor. You may keep your copy of the questions.*

**NO CALCULATORS  
75 MINUTES**

1. Five candidates ran for the office of dog catcher. No two had the same number of votes. The winner had 10 votes. What is the largest possible number of votes for the last place candidate?  
a. 3 b. 4 c. 5 d. 6 e. 7
2. How many of the numbers 555555, 5555555, 55555555, 555555555 and 5555555555 are divisible by 9?  
a. 0 b. 1 c. 2 d. 3 e. None of the preceding.
3.  $A=7/8$ ,  $B=66/77$ ,  $C=555/666$ ,  $D=4444/5555$ ,  $E=33333/44444$ . Which is the largest?  
a. A b. B c. C d. D e. E
4. An old fashioned toaster can toast one side of up to 4 slices of bread in one minute. What is the least time required to toast both sides of 9 slices?  
a. 4 b. 5 c. 6 d. 7 e. None of the preceding.
5. In a restaurant a certain main course costs \$22.50 more than the dessert. The main course and the dessert together cost 12 times the price of the dessert. The cost in dollars of dessert is in the range of  
a. (1.90, 2.10) b. (2.10, 2.30) c. (2.30, 2.50) d. (2.50, 4.00)  
e. more than 4.00

6. The largest integer  $n$  so that  $8^n$  divides  $44^{44}$  is  
 a. 8 b. 22 c. 29 d. 44 e. 88
7. Order the following 4 numbers from smallest to largest:  
 $A = (-2)^{-2}, B = (-3)^{-3}, C = (-4)^{-4}, D = -\sqrt[7]{77}$   
 a. DBCA b. ABCD c. BCAD d. DABC e. None of the preceding.
8. Five girls A,B,C,D,E sit on 2 chairs and 3 stools, each seating exactly one girl. Who sits on the chairs if A and B sit on the same type of seat, B and D sit on a different type, D and E sit on a different type?  
 a. CD b. AD c. BE d. BC e. AB
9. A (12-hour) wristwatch is slow. It loses 5 minutes per day. Assuming that it is now set to the correct time, how long will it be before it next shows the correct time?  
 a. 2400 minutes b. 60 hours c. 80 days d. 144 days e. 240 days
10. For which value of  $a$  does the straight line  $y = 6x$  intersect the parabola  $y = x^2 + a$  in exactly one point?  
 a. 7 b. 8 c. 9 d. 10 e. 11
11. Let  $a = \log 3$  and  $b = \frac{\log 3}{\log(\log 3)}$  (logarithms are to base 10). The number  $a^b$  is equal to  
 a.  $1/\log 3$  b. 1 c.  $\log 3$  d.  $1/3$  e. 3
12. Let  $f(x) = (x + a)^3 + b$ . How many pairs of real numbers  $(a, b)$  are there such that  $f(0) = 1$  and  $f(1) = 2$ ?  
 a. 0 b. 1 c. 2 d. 3 e. 4
13. Jack and Jill went up the hill at 4 mph. They started tumbling down at 6 mph. Unfortunately, they hit a rock (and broke their crowns) at exactly halfway down the hill. What was their average speed in mph during the trip up and halfway down?  
 a. 4.5 b.  $\frac{14}{3}$  c. 4.8 d. 5 e. 5.4



14. To move a heavy marble slab ancient temple builders placed cylindrical rollers underneath the slab as shown above. Each roller has a radius of 5 inches. The cylinders go through one rotation. The distance in inches the slab moves is in the interval
- a. (15, 30)   b. (30, 45)   c. (45, 60)   d. (60, 75)  
 e. None of the preceding.
15. If ABCDE is a regular pentagon, then
- a.  $0^\circ < \angle ACE \leq 15^\circ$    b.  $15^\circ < \angle ACE \leq 30^\circ$    c.  $30^\circ < \angle ACE \leq 45^\circ$   
 d.  $45^\circ < \angle ACE \leq 60^\circ$    e.  $60^\circ < \angle ACE < 90^\circ$
16. If  $n$  is the number of integers between 1 and 999 that have at least one 7 in their decimal representation, then
- a.  $75 < n \leq 125$    b.  $125 < n \leq 175$    c.  $175 < n \leq 225$   
 d.  $225 < n \leq 275$    e.  $275 < n \leq 325$
17. Dumbbells weigh 20, 30 or 40 lbs. The total weight of a pile of dumbbells is 800 lbs. The number of dumbbells in the pile that weigh 30 lbs can NOT be
- a. 2   b. 3   c. 4   d. 6   e. 10
18. The digits 1, 9, 9, 8 in 1998 have their total  $1+9+9+8=27$ . The next time the sum of the digits is 27 happens between the years
- a. 2500 and 2700   b. 2701 and 2900   c. 2901 and 3100  
 d. 3101 and 9900   e. 9901 and 9999

19. What is the largest number of pieces into which the region  $y \geq x^2$  can be cut by 4 straight lines?  
 a. 8 b. 9 c. 10 d. 11 e. None of the preceding.

20. The number  $\sqrt{7 + 4\sqrt{3}} + \sqrt{7 - 4\sqrt{3}}$  is closest to which of the following:  
 a. 3.6 b. 3.7 c. 3.8 d. 3.9 e. 4.0

21. Consider the triangular array

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1
2 3
4 5 6
7 8 9 10
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.....

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The sum of the elements in the 100th row is

- a. 1000100 b. 1000000 c. 500000 d. 500050 e. 5000050
22. A train travels at speed  $s_1$  and a runner runs at speed  $s_2$ . It takes  $t_1$  seconds for the entire train to pass the runner when they are going in the same direction and  $t_2$  seconds when they are going in opposite directions. Suppose  $s_1/s_2 = t_1/t_2$ . Then  $s_1/s_2$  is closest to  
 a. 3 b. 2.5 c. 2 d. 1.5 e. 1
23. A quadratic polynomial  $p(x)$  satisfies  $p(0) = 3, p(1) = 5, p(2) = 8$ . Then  $p(5)$  is  
 a. 22 b. 23 c. 24 d. 25 e. None of the preceding.
24. The angle between the hour hand and the minute hand is measured every minute, beginning at 12:01 and ending at 11:59. The smallest angle observed is  
 a.  $5.5^\circ$  b.  $3^\circ$  c.  $1^\circ$  d.  $0.5^\circ$  e.  $0.25^\circ$
25. The length of one edge of a regular tetrahedron inscribed in a sphere of unit radius is  
 a.  $\sqrt{3}$  b.  $\frac{\sqrt{6}}{2}$  c.  $\frac{\sqrt{3}}{2}$  d.  $\frac{2\sqrt{2}}{3}$  e.  $\frac{2\sqrt{6}}{3}$