UNIVERSITY OF MARYLAND MATHEMATICS COMPETITION

PART I, 1999

No calculators are allowed. 75 min.

For each of the following questions, carefully blacken the appropriate box on the answer sheet with a #2 pencil. Do not fold, bend or write stray marks on either side of the answer sheet. Each correct answer is worth 4 points. Two points are deducted for each incorrect answer. Zero points are given if no box, or more than one box, is marked. Note that wild guessing is apt to lower your score.

1. A father and son have the same birthday, October 20. Today the father becomes 42 and the son becomes 11. In what year will the father be exactly twice as old as the son?
2009, 2011, 2013, 2017, 2019

2. A man works for 10 days. On the first day he is paid $2. On the second day, $4. On the third day, $8. On each successive day, his pay is doubled.
How much is he paid altogether for the 10 days?
1023, 1999, 2000, 2046, 2048

3. Find the smallest integer N so that $|N-9|<3$.
5, 6, 7, 11, 12

4. If $3x+4y=10$ and $2x+7y=11$, then x is
-1, 0, 1/2, 1, 2

5. Donald Duck can eat 2 pizzas in 3 minutes, while Goofy can eat 3 pizzas in 2 minutes. At these rates, how many pizzas can they eat together in an hour?
54, 96, 130, 216, 250

6. There are 37 red, green, blue, and yellow marbles in a bag. There are 3 more red marbles than green marbles, 2 more red than blue, and 4 more yellow than blue. How many green marbles are there?
5, 7, 9, 11, 12

7. Order the following numbers from smallest to largest:
a) 2^{1999}, b) 1999^2, c) $10^{\log 2^{1999}}$.
bac, bca, cba, abc, none of the preceding

8. Suppose a positive integer N is divisible by 21 and by 9. What is the smallest possible number of positive integers that divide N?
3, 4, 5, 6, 7

9. The freshmen at Wisdom High School are required to take exactly 2 of the following 3 courses: English, Math, Social Studies. The freshmen enrollments are: 20 in English, 17 in Math, 11 in Social Studies. How many freshmen are there?
24, 25, 26, 27, 48

10. A cow, a horse, and a goat went out for lunch. The cow ate 5 squares of grass, each of side 6 yards. The horse ate 3 circles, each of radius 5 yards. The goat ate an equilateral triangle of side 22 yards. Order the animals from the smallest area to largest area eaten.
cow,horse,goat; cow,goat,horse; horse,cow,goat; horse,goat,cow; goat,horse,cow

11. The last digit of 777^{777} is
1, 3, 5, 7, 9

12. Superman and Batman (working together) peel a bucket of potatoes in 20 minutes, Superman and Cinderella in 15 minutes, Cinderella and Batman in 12 minutes.
How long does it take Superman to peel a bucket of potatoes?
1 hour, 45 minutes, 40 minutes, 30 minutes, none of the preceding

13. Snoopy must choose among the following three long-distance plans for his doghouse.
Plan A: 99 cents for any call up to 20 minutes, plus 5 cents for each additional minute over 20;
Plan B: a flat rate of 10 cents per minute;
Plan C: a flat rate of 8 cents per minute after a 25 cents connection charge for each call.
Suppose 10% of Snoopy's calls are 1 minute long, 10% are 5 minutes, 30% are 10 minutes, 30% are 20 minutes, 20% are 30 minutes.
Order the plans from cheapest to most expensive:
ABC, BAC, BCA, ACB, CBA

14. How many 3-letter sequences can be made using the letters in the word "BOOKKEEPER"?
(For example, "OKO" is one acceptable sequence, "OPP" is not an acceptable sequence.)
36, 54, 166, 216, 1999

15. Mickey Mouse (M) and Donald Duck (D) can paddle a canoe together in still water at 5 mph, M alone paddles at 2 mph, D alone paddles at 3 mph.
At 12 noon they start paddling together down the river, which flows at 1 mph.
At 1 pm D (who was in the back) lost his paddle but did not tell M.
At 2 pm M looked back, saw that D did not have a paddle and punched him, which made

D lose his straw hat. Then M gave his paddle to D who paddled upstream to the lost paddle.
Immediately after retrieving the paddle, they turned downstream and paddled together
until they reached D's hat at
2:32 pm, 2:58 pm, 3:04 pm, 3:22 pm, none of the preceding
Note: Both the lost paddle and the hat floated with the current.

16. A responsible cow goes to the same field each day at 6 am, eats the same fraction
of the available grass, fertilizes the field and goes back home at noon. The grass
grows only from noon to 6 am the next day. There is three times as much
grass at 6 am each day as there was at noon the previous day. What fraction of the grass
must the cow eat to avoid both starvation and overproduction?
3/4, 1/4, 1/3, 1/2, none of the preceding

17. Let $g=10^{100}$ (googol) and $G=10^g$ (googolplex). In which interval
does googol factorial $g!=1\cdot2\cdot3\cdot\ldots$ 10^{100} lie?
$g<g!<G$, $G<g!<10G$, $10G<g!<10^G$, $10^G<g!$, none of the preceding

18. What is the largest postage that cannot be paid exactly with an unlimited supply of 6-cent
and 7-cent stamps?
15, 29, 32, 41, 43

19. Let r, s, t be the roots of x^3-7x^2+8x+2. What is the value
of $1/r+1/s+1/t$?
-4, -7/2, -2, 2, 8

20. Let f be a function defined for the positive integers such that for every positive integer n,
(i) f(n) is a positive integer, (ii) f(n+1)>f(n), and (iii) f(f(n))=f(n).
How many such functions are there?
0, 1, 2, 12, infinitely many

21. Given a list of one million different numbers, a computer puts them in
increasing order as follows:
(i) assign n=0;
(ii) increase n by one;
(iii) if the nth number is less than the next one, go to (iv); otherwise modify the list
by interchanging the n-th and (n+1)st numbers and go to (i);
(iv) if $n<10^6-1$, go to (ii); otherwise stop.
The number M of times the computer performs operation (ii) depends on the initial list.
In which interval does the maximal value of M (among all lists) lie?
$M<10^7$, $10^7<M<10^{10}$, $10^{10}<M<10^{15}$, $10^{15}<M<10^{20}$, $10^{20}<M<10^{25}$

22. The rows of an 8 by 8 checkerboard are marked a through h from top to bottom;
the columns are numbered 1 through 8 from left to right (the northwest corner is 1a,
the southeast corner is 8h). Depending on which two squares are removed, decide
if it is possible to cover (without overlaps) the remaining figure with thirty-one 2 by 1
or 1 by 2 dominoes (rectangles).
1) 1a and 8a, 2) 1a and 8b, 3) 1a and 8h.
yes, yes, yes; yes, no, no; no, yes, no; no, no, yes; no, no, no

23. In triangle ABC, the point D lies on BC, and AD is the bisector of angle BAC.
If |AB|=c, |AC|=b, and angle CAD=w, then |AD| is
(bc sinw)/(b+c), (b cosw+c sinw)/2, (b sin2w + c cos2w)/2, (2bc cosw)/(b+c), 2bcsinw/(b+c)

24. Let a polynomial $P(k)=a_0k^4+a_1k^3+a_2k^2+a_3k+a_4$ satisfy $P(0)=P(1)=P(2)=P(-1)=0$
and P(-2)=12. Then P(3) equals
1/3, -1/2, 1, 2, 12

25. Three spheres of radius 1 are pairwise tangent and resting on a horizontal table. A fourth
sphere (also of radius 1) is placed on top with its center above the center of the triangle
formed by the 3 spheres. The height of the center of the top sphere above the table is
$3^{1/2}$, $1+3^{1/2}$, $(2\ 2^{1/2}+3^{1/2})/3^{1/2}$, $(2^{1/2}+3^{1/2})/2^{1/2}$, none of the preceding

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