

PART I, 2000

No calculators are allowed. 75 min.

For each of the following questions, carefully blacken the appropriate box on the answer sheet with a #2 pencil. Do not fold, bend or write stray marks on either side of the answer sheet. Each correct answer is worth 4 points. Two points are deducted for each incorrect answer. Zero points are given if no box, or more than one box, is marked. Note that wild guessing is apt to lower your score.

1. Which of the following the following is the largest?

- a. 2^{3^4} , b. 4^{3^2} , c. 8^{4^2} , d. 16^{8^2} , e. 8^{4^2}

2. Rank the following collections of round pizzas in increasing order of area:

(A) five 10-inch pizzas; (B) three 14-inch pizzas; (C) two 16-inch pizzas.

- a. BAC, b. CAB, c. CBA, d. ABC, e. ACB

3. A computer virus is eating disk space. During the first day it eats $\frac{1}{2}$ of the disk space. During the second day it eats $\frac{1}{3}$ of the remaining disk space. The third day it eats $\frac{1}{4}$ of what still remains and the fourth day it eats $\frac{1}{5}$ of what is left. What fraction of the original disk space remains uneaten?

- a. $\frac{1}{5}$, b. $\frac{1}{6}$, c. $\frac{1}{10}$, d. $\frac{1}{12}$, e. $\frac{1}{24}$

4. Walter Wildcat runs 3 times as fast as Timothy Terrapin walks. Suppose that Timothy, who is smarter than Walter, finishes an exam at 2:00 and begins walking home. Walter finishes at 2:12 and runs after Timothy. At what time does Walter catch up to Timothy?

- a. 2:15, b. 2:16, c. 2:17, d. 2:18, e. 2:19

5. An integer p is prime if $p \geq 2$ and the only divisors of p are 1 and p . Let M be the product of the first 100 primes. How many 0's are at the end of M ?

- a. 0, b. 1, c. 10, d. 20, e. 100

6. Five sheep can eat the grass of 5 football fields in 5 days. How many days would it take 3 sheep to eat the grass of 3 football fields?

- a. 2, b. 3, c. 4, d. 5, e. 6

7. Three little pigs open e-trade accounts and become day traders. After hard work they achieve the following steady rates of losing money: the first one loses \$1000 an hour, the second one loses \$1000 in two hours, and the third one loses \$1000 in three hours. The number of minutes it takes all three of them together to lose \$2000 is between

- a. 180 and 270, b. 120 and 180, c. 90 and 120, d. 66 and 90, e. 60 and 66

8. The grand prize in a basketball tournament is the largest collection of the new golden dollar coins that can be placed on a 50 by 94 foot basketball court without overlapping. The new dollar coin is 1.04 inches in diameter. The prize is worth between

- a. \$10 and \$10,000, b. \$10,000 and \$40,000, c. \$40,000 and \$200,000, c. \$200,000 and \$1 million, d. \$1 million and \$10 million

9. In triangle ABC, angle C is three times as big as angle A and angle B is twice as big as angle A. The ratio of the length of AB to BC is

- a. $\frac{1}{3}$, b. $\frac{1}{2}$, c. $\frac{2}{3}$, d. $3^{1/2}$, e. 2

10. Of the seven points (9,17), (6,11), (3,5), (7,12), (3.5,6), (5,10), (5,9), five lie on a straight line. Which two do NOT lie on that line?

- a. (5,10) and (7,12), b. (3,5) and (5,9), c. (9,17) and (7,12), d. (6,11) and (3,5), e. (3.5,6) and (5,9)

11. Let a, b, c be nonzero integers. Let $p(x) = ax^2 + bx + c$. Which of the following CANNOT occur?

- a. $p(x)$ has only one root and that root is positive
 b. $p(x)$ has only one root and that root is negative
 c. $p(x)$ has only one root and that root is rational
 d. $p(x)$ has only one root and that root is irrational
 e. $p(x)$ has no real roots

12. How many pairs of positive integers (a, b) are there with $a \leq b$ and $\frac{1}{6} = \frac{1}{a} + \frac{1}{b}$?

- a. 1, b. 2, c. 3, d. 4, e. 5

13. Let A be the area of a circle that has radius $(\pi)^{1/2}$. Within which interval does A lie?

- a. $0 < A < 1$, b. $1 < A < 4$, c. $4 < A < 9$, d. $9 < A < 16$, e. $16 < A < 25$

14. Ten people play a round-robin chess tournament (every person plays everyone else exactly once). In each game, the winner gets 3 points, the loser gets 0 points, and in case of a draw (tie), each of the two players gets 1 point. The total number of points given to all players is 130. How many games are draws?

- a. 1, b. 2, c. 3, d. 4, e. 5

15. Berries are 90% water and 10% pulp, measured by weight, when freshly picked. If the berries sit for a week, part of the water evaporates and the berries become 80% water and 20% pulp. If 20 pounds of freshly picked berries sit for a

week, how much will they weigh?
a. 8 pounds b. 10 pounds c. 12 pounds d. 15 pounds e. 18 pounds

16. The lower two vertices of a square lie on the x -axis and the upper two vertices of the square lie on the parabola $y=15-x^2$. What is the area of the square?

a. 9, b. $10(2)^{1/2}$, c. 16, d. 25, e. 36

17. The sum of several (not necessarily different) positive integers is 10. What is the largest possible value of their product?

a. 25, b. 32, c. 36, d. 54, e. 64

18. Order $\sin 1$, $\sin 2$, $\sin 3$ from smallest to largest (the angles are measured in radians).

a. $\sin 1 < \sin 2 < \sin 3$, b. $\sin 3 < \sin 2 < \sin 1$, c. $\sin 1 < \sin 3 < \sin 2$, d. $\sin 2 < \sin 1 < \sin 3$, e. $\sin 3 < \sin 1 < \sin 2$

19. The lines AB and CD are parallel and a distance 4 apart. Suppose that AD intersects BC at a point P between the lines. If $AB=4$ and $CD=12$, how far is P from the line CD ?

a. 1, b. 2, c. 3, d. 4, e. 6

20. The triangular numbers are the numbers of the form $n(n+1)/2$, where $n=1, 2, 3, \dots$. How many of the first 100 triangular numbers end with a 0?

a. 10, b. 20, c. 30, d. 40, e. 50

21. Let a and b be real numbers with $a > 1$ and $b > 1$. Let $\log_a(b)$ denote the logarithm of b for the base a . Four students compute the product $\log_a(b) \cdot \log_b(a)$. The first gets the answer 1. The second gets $\log_{ab}(ab)$. The third gets $a^b b^a$. The fourth gets $\log_b(a) + \log_a(b)$. How many of the students are correct?

a. 0, b. 1, c. 2, d. 3, e. 4

22. A student writes $44 \cdot 55 = 3506$. The student is calculating in base b and has written a correct equation. What is b ?

a. 7, b. 8, c. 9, d. 12, e. 14

23. Horace always lies. One day he told his neighbor Frank "At least one of us never lies." From this information it is certain that

a. Frank always lies. b. Frank sometimes lies. c. Frank always tells the truth. d. Frank sometimes tells the truth. e. Frank has never said anything.

24. Suppose $f(x+1, y) = f(x, y) + y + 1$, $f(x, 0) = x$, $f(x, y) = f(y, x)$ for all x, y . What is $f(12, 5)$?

a. 5, b. 6, c. 12, d. 17, e. 77

25. At 1:00 pm on October 25, 2000, a dog falls asleep. It sleeps for 5 hours, then is awake for 5 hours, then sleeps for 5 hours, etc. At 1:17 pm on the same day, a cat falls asleep. It sleeps for 2 hours, then is awake for 2 hours, then sleeps for 2 hours, etc. Determine the largest integer H so that in every unbroken 24 hour period in the year 2001 there must be a total of at least H hours during which both animals will be asleep.

a. 4, b. 5, c. 6, d. 7, e. 10