MATHEMATICS COMPETITION
UNIVERSITY OF MARYLAND
1998 PART II

Wednesday, December 2, 1998, 1 p.m.–3 p.m.

Use SEPARATE sheets for different problems. Submit only the material you want graded. Each problem is worth 30 points. No calculators are allowed.

PROOFS MUST BE GIVEN FOR ALL ANSWERS

1. Four positive numbers are placed at the vertices of a rectangle. Each number is at least as large as the average of the two numbers at the adjacent vertices. Prove that all four numbers are equal.

2. The sum 498+499+500+501=1998 is one way of expressing 1998 as a sum of consecutive positive integers. Find all ways of expressing 1998 as a sum of two or more consecutive positive integers. Prove your list is complete.

3. An infinite strip (two parallel lines and the region between them) has a width of 1 inch. What is the largest value of A such that every triangle with area A square inches can be placed on this strip? Justify your answer.

4. A plane divides space into two regions. Two planes that intersect in a line divide space into four regions. Now suppose that twelve planes are given in space so that a) every two of them intersect in a line, b) every three of them intersect in a point, and c) no four of them have a common point. Into how many regions is space divided? Justify your answer.

5. Five robbers have stolen 1998 identical gold coins. They agree to the following: The youngest robber proposes a division of the loot. All robbers, including the proposer, vote on the proposal. If at least half the robbers vote yes, then that proposal is accepted. If not, the proposer is sent away with no loot and the next youngest robber makes a new proposal to be voted on by the four remaining robbers, with the same rules as above. This continues until a proposed division is accepted by at least half the remaining robbers. Each robber guards his best interests: He will vote for a proposal **if and only if** it will give him **more** coins than he will acquire by rejecting it, and the proposer will keep as many coins for himself as he can. How will the coins be distributed? Explain your reasoning.