UNIVERSITY OF MARYLAND MATHEMATICS COMPETITION

PART II, 1999

1. Twelve tables are set up in a row for a Millenium party. You want to put 2000 cupcakes on the tables so that the numbers of cupcakes on adjacent tables always differ by one (for example, if the 5th table has 20 cupcakes, then the 4th table has either 19 or 21 cupcakes, and the 6th table has either 19 or 21 cupcakes).
   a) Find a way to do this.
   b) Suppose a Y2K bug eats one of the cupcakes, so you have only 1999 cupcakes. Show that it is impossible to arrange the cupcakes on the tables according to the above conditions.

2. Let P and Q lie on the hypotenuse AB of the right triangle CAB so that $|AP|=|PQ|=|QB|=|AB|/3$. Suppose that $|CP|^2+|CQ|^2=5$.
   Prove that $|AB|$ has the same value for all such triangles, and find that value.
   Note: $|XY|$ denotes the length of the segment XY.

3. Let P be a polynomial with integer coefficients and let a, b, c be integers. Suppose $P(a)=b$, $P(b)=c$, and $P(c)=a$.
   Prove that $a=b=c$.

4. A lattice point is a point (x,y) in the plane for which both x and y are integers. Each lattice point is painted with one of 1999 available colors.
   Prove that there is a rectangle (of nonzero height and width) whose corners are lattice points of the same color.

5. A 1999-by-1999 chocolate bar has vertical and horizontal grooves which divide it into 19992 one-by-one squares.
   Caesar and Brutus are playing the following game with the chocolate bar: A move consists of a player picking up one chocolate rectangle; breaking it along a groove into two smaller rectangles; and then **either** putting both rectangles down **or** eating one piece and putting the other piece down. The players move alternately. The one who cannot make a move at his turn (because there are only one-by-one squares left) loses.
   Caesar starts.
   Which player has a winning strategy?
   Describe a winning strategy for that player.

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