UNIVERSITY OF MARYLAND MATHEMATICS COMPETITION

PART II, 2000

1. There are 2000 cans of paint. Show that at least one of the following two statements must be true.
   - There are at least 45 cans of the same color.
   - There are at least 45 cans all of different colors.
2. The measures of the 3 angles of one triangle are all different from each other but are the same as the measures of the 3 angles of a second triangle. The lengths of 2 sides of the first triangle are different from each other but are the same as the lengths of 2 sides of the second triangle.
   Must the length of the remaining side of the first triangle be the same as the length of the remaining side of the second triangle? If yes, prove it. If not, provide an example.
3. Consider the sequence $a_1=1, a_2=2, a_3=5/2, \ldots$ satisfying $a_{n+1}=a_n+(a_n)^{-1}$ for $n>1$. Show that $a_{10000}>141$.
4. Prove that no matter how 250 points are placed in a disk of radius 1, there is a disk of radius 1/10 that contains at least 3 of the points.
5. Prove that:
   Given any 11 integers (not necessarily distinct), one can select 6 of them so that their sum is divisible by 6.
   Given any 71 integers (not necessarily distinct), one can select 36 of them so that their sum is divisible by 36.

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