2000 SOLUTIONS, PART II

1. If there are at most 44 colors and and most 44 cans of the same color, then the total number of cans is 44·44=1936. This is a contradiction.
2. The answer is NO. Here is an example. The sides of the first triangle are 1, 1.5, 1.5·1.5=2.25. The sides of the second triangle are 1.5, 1.5·1.5=2.25, 1.5·1.5·1.5=3.375. In both cases the triangle inequality is satisfied, so the triangles exist. The triangles are similar. The measures of the 3 angles of the first triangle are all different from each other but are the same as the measures of the corresponding angles of the second triangle.
3. $(a_{n+1})^2=(a_n)^2+2+(a_n)^{-2} >(a_n)^2+2$. Therefore, by induction, $(a_n)^2>2n$ for all n>2, so $(a_{10000})^2>20000>141^2$, so $a_{10000}>141$.
4. Consider the 250 disks, each of radius 1/10 that are centered at each of the points. The sum of the areas of these disks is 2.5pi, and the union of the disks is contained inside a disk of radius 1.1. Since $2.5pi>2(1.1)^2pi$, there is a point P (not necessarily in the chosen set) that is contained in at least 3 of the small disks. Thus, the disk of radius 1/10, centered at P contains at least three of the original points.
5. a. Given any five integers, either three of them have the same remainders when divided by 3 or three of them have all different remainders. In both cases, the sum of these three is a multiple of 3, say 3a. Take any five of the remaining 8=11-3 integers and select three with the sum 3b. Of the remaining 5=8-3 integers select three with the sum 3c. Two of the integers a, b, c are of the same parity, say, a and b. The sum 3a+3b= 3(a+b) is divisible by 6.
   b. Given 71 integers, select six with the sum $6a_1$ and repeat the procedure ten more times, every time at least 11 integers being available. We obtain eleven pairwise disjoint sixtuples with the sums $6a_1$,..., $6a_{11}$. Of the eleven integers, a_1 through a_{11}, we can select six, say, a_1 through a_6, with the sum divisible by 6. The six sixtuples with the sums $6a_1$,..., $6a_6$ are the required 36 integers with the sum $6(a_1+...+a_6)$ divisible by 36.