## UNIVERSITY OF MARYLAND MATHEMATICS COMPETITION

# PART II, 2001

UNIVERSITY OF MARYLAND HIGH SCHOOL MATHEMATICS COMPETITION
PART II
December 5, 2001, 1:00--3:00
NO CALCULATORS
2 hours

1. A band of pirates unloaded some number of treasure chests from their ship. The number of pirates was between 60 and 69 (inclusive). Each pirate handled exactly 11 treasure chests, and each treasure chest was handled by exactly 7 pirates. Exactly how many treasure chests were there? Show that your answer is the only solution.

2. Let a and b be the lengths of the legs of a right triangle, let c be the length of the hypotenuse, and let h be the length of the altitude drawn from the vertex of the right angle to the hypotenuse. Prove that $c+h>a+b$.

3. Prove that
$1/70< (1/2)(3/4)(5/6)...(2001/2002) < 1/40$.

4. Given a positive integer $a_1$ we form a sequence $a_1$, $a_2$, $a_3$ ... as follows: $a_2$ is obtained from $a_1$ by adding together the digits of $a_1$ raised to the 2001 st power; $a_3$ is obtained from $a_2$ using the same rule, and so on. For example, if $a_1 =25$, then $a_2 =2^{2001}+5^{2001}$, which is a 1399-digit number containing 106 0's, 150 1's, 124 2's, 157 3's, 148 4's, 141 5's, 128 6's, 150 7's, 152 8's, 143 9's, So

$a_3 = 106 \times 0^{2001}+ 150 \times 1^{2001}+ 124 \times 2^{2001}+ 157 \times 3^{2001}+...+ 143 \times 9^{2001}$
which is a 1912-digit number, and so forth. Prove that if any positive integer $a_1$ is chosen to start the sequence, then there is a positive integer M (which depends on $a_1$) that is so large that $a_n < M$ for all n=1,2,3,...

5. Let P(x) be a polynomial with integer coefficients. Suppose that there are integers a, b, and c such that P(a)=0, P(b)=1, and P(c)=2. Prove that there is at most one integer n such that P(n)=4.